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ABSTRACT

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Abstract

The logic of set theory is applied to the problem of infrastructure and transition in developmental analysis. The analysis necessitates the decomposition of stages into components. The decomposition generates sets within sets, and from a sequential perspective, progressions within progressions. The relation of lower to higher, and precursor to successor items varies from the trivial to the rigorous. In a trivial analysis, a precursor is neither necessary nor sufficient for a successor; in a rigorous analysis, a precursor is necessary and sufficient. Infrastructure and transition are interdependent. When components are common to successive stages, stages are transformed or correlatively transformed; when components are discrete, stages are substituted, added, or deleted; and when some components are common and some discrete, stages are hybrid. Any transformation or correlative transformation implies a correspondence, integration, or differentiation with parallelism, subordination, or emergence. Any substitution, addition, or deletion implies a coincidence, augmentation, or reduction with coplanarity, expansion, or contraction. The implications of this analysis are discussed in relation to developmental theory and theory evaluation.

Infrastructure and Transition in Developmental Analysis

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Educational Testing Service

A Potpourri of Principles

An amateur's guidebook to the description of developmental structure might include some of all of the following terms: assimilation, accommodation, centralization, integration, hierarchization, coordination, incorporation, subordination, differentiation, domination, and globalization. Provided with this verbal armamentarium, he is prepared for contemplation of general formulations such as Werner's Orthogenic Principle, "[Development] proceeds from a state of relative globality and lack of differentiation to a state of increasing differentiation, articulation, and hierarchic integration [1957, p. 126]." In some sense, from Werner's perspective, the more complex is the more mature.

However, in a domain where principle appears to rule by fiat, an Un-Wernerian Orthogenic Principle may seem equally plausible, "[Development] proceeds from a state of relative elementalism and differentiation to a state of increasing globalization, dedifferentiation, and simplification." That is, the more simple is the more mature.

Perhaps there is a germ of truth in both principles and they might be rewritten as the synthetic principle, "[Development] proceeds from a state of relative globality and differentiation, to a state of increasing articulation, simplification and hierarchic integration." Although this collection of principles may seem a disrespect to the grand theoretical orientation, it is meant to illustrate the perplexity inherent in the description and understanding of developmental structure.

Werner's biologism. Psychological theories which rely on nonformal biological metaphor share this difficulty: A precise analog of descriptive statement is absent. Globality, differentiation, articulation, and other kindred terms possess a descriptive justification so long as the substrate to which these terms refer is unambiguous. But when structure is ambiguous and the substrate complex as in developmental psychological analysis, the Wernerian, Un-Wernerian, or synthetic principles appear, a fortiori, equally plausible. The problem is that all might be true with reference to aspects of structure or structure differently conceived. Principles without clear descriptive reference serve as a theoretician's Rorschach.

Piaget's constructivism. Psychological theories which provide a formal analog to embryological development, such as Piaget's constructivism, deal with only a subset of the plausible forms of change. From Piaget's perspective, a structure is a system of transformations (Piaget, 1970a, 1971). The structure is "preserved or enriched" by the interplay of its transformations "which never go beyond its frontiers, nor employ elements that are external to it [Wilden, 1973, p. 311]." As Wilden argues, a Piagetian structure is therefore a closed structure, "a system closed under transformation [Piaget, 1970b, p. 6]."

Even if it is admitted that the operations which organize structure in Piaget's sense are extremely general, and all that Piaget's constructivism argues is the necessity of a consistent application of rules of formation and regulation from stage to stage, the theory, nevertheless, excludes the substitution or replacement of structures. Stages follow one another in a state-determined way. Given the operation of the system invariants, the "stronger" structure necessarily follows from the "weaker" structure in a fixed

sequence (Piaget, 1970b, p. 141). With Piaget, substantive discontinuity is inadmissible. But it is for precisely this reason that Piaget's constructivism deals with only a subset of forms of change because substantive discontinuity is plausible, and within a more general framework, admissible.

Flavell's eclecticism. Recently, Flavell has proposed a classification of the general forms of change which might apply to the process of development (Flavell, 1970, 1972; Flavell & Wohlwill, 1969). Since the classification provides for the addition and substitution of items of analysis (substantive discontinuities) the classification is, in this sense, more general than that proposed by Piaget. However, beyond this general comparison, there is little basis of similarity between Piaget and Flavell. Piaget is a deductive formalist; Flavell is an inductive empiricist. While Piaget's particular brand of formalism generates a too narrow perspective of the forms of admissible change, Flavell's eclecticism provides a set of categories linked to no explicit theory of structure. As Flavell summarizes, "...the categories finally arrived at have simply not proven to be the unitary, nonoverlapping, definitionally-elegant affairs originally hoped for [1972, p. 9]."

A proposed formalism. In an earlier paper (Van den Daele, 1969) the topography of stage sequences was described through application of a set of assertions. The term "topography" is borrowed because little attention was given to the infrastructure of elements in a stage sequence. The discussion of elements was limited to their most general features. In the present paper, a formalism for the description of organization and change in infrastructure is proposed. The aim of the formalism is to provide a frame of reference, a consistent nomenclature, and a representation of the plausible forms of

transition which may characterize a stage sequence. In contrast to a topography of stages, an anatomy of stage organization and change is generated.

Infrastructural Analysis

The unit of developmental infrastructural analysis is the unitary sequence. A unitary sequence is an ordered set of elements associated in some way. In psychological analysis, the unitary sequence is the set of states or stages associated with an individual or group of individuals of the same class for whom the sequence is descriptive. The unitary sequence is the simplest subset of a multiple progression. Any multiple progression may be partitioned into a set of unitary sequences (Figure 1).

Insert Figure 1 about here

Domain, perspective, and boundary conditions. The elements which define any progression are members of a domain. A domain is an area of discourse, a universe of interest, a bounded content. The limits of a domain are definitional and may vary from the general, as in a mathematical inquiry about a theory of sets, to the specific, as in an ethological evaluation about a species-specific action pattern.

A domain may be viewed from any number of perspectives. A perspective is an orientation or mode of analysis applied to a domain. An orientation implies some set of principles, rules, or algorithms applied to the items of analysis and their relationships. A perspective is well formed or fuzzy. A well-formed perspective is circularly consistent; a fuzzy perspective is inconsistent.

The perspective applied to a domain determines the boundary conditions of analysis or the set of items proper to a sequence. A consistent analysis applies the same perspective to all potential items. If one perspective is applied to one item, then another perspective to another item, the analysis lacks coherence.

Elements and order. Elements of a set may be ordered by empirical or formal criteria. Empirical criteria require evidence that the order of elements correspond to some chronological or maturational order. Formal criteria require that the order correspond to some rule of succession. Empirical criteria are independent, and formal criteria are dependent upon the organization of elements.

Empirical criteria presume change in response probability associated with some measure of maturity. An observed order of response defines an order of elements. Formal criteria presume an order of elements correspond to a form of infrastructure and mode of transition.

Decomposition. From the perspective of stage infrastructure, the elements of a progression are analogous to the tip of an iceberg. They provide only the most cursory description of structure. The elements of a progression may represent organizations of any desired complexity. The elements of an ordered set may be decomposed into any number of components. A component is an aspect, part, subspace, or any rational differentia of an element.

If the initial elements of a sequence are designated the first order of analysis, then $1 + n$ orders of analysis may be obtained through the decomposition of components of the nth order where the nth order is the preceding level of analysis. Insofar as elements may contain components, and these components, subcomponents, and so on, levels generated through iteration or repetition of this procedure are hierarchically ordered.

Components so obtained map the infrastructure of an element. The relations of components within elements are synchronic, but like the diachronic representation of multiple models, map various types of partial and complete parallelism, divergence, and convergence (Riegel, 1973; Van den Daele, 1969) (Figure 2).

Insert Figure 2 about here

At some level the components derived at the nth + 1 iteration may be identical to the components obtained at the nth iteration. Although iteration may continue indefinitely, when successive iteration yields identical components, levels obtain a lower bound. Iteration provides no new information. If then, the nth iteration obtains an empty set, since an empty set may contain only an empty set, the iteration may be terminated. In general, a sufficiently repeated iteration obtains a lower bound.

Types of decomposition. The task of partitioning an element into a set of components and subcomponents derives through the application of some set of principles, decision-rules, or algorithms (a perspective). Whatever principles are adopted for an analysis constrain the definition and relation of components and determine what are the proper subsets of an element. While the principles which determine the constitution of components define an indefinitely large set, these principles may be classed into four types dependent upon the restrictions placed upon the relation between an element and its components.

A type I analysis specifies a set of components of an element which are neither necessary nor sufficient for the presence of the element. The

relation between one level of analysis and another is capricious. A higher level does not constrain a lower level or vice versa. Correspondingly, the decision rules which belong to this group are weak and provide no procedure for the identification of what is arbitrary or nonarbitrary. An example of this type of analysis occurs in early dogma about maternal impressions and fetal development (Riddell, 1909).

A type II analysis partitions an element into a set of components which are sufficient, but not necessary, for the presence of the element. While the type I analysis is an arbitrary mapping of components, the type II analysis provides an "empirical groping" toward a more adequate identification of components. In a type II analysis, any number of components may be identified so long as these are associated with the element. In this sense, the approach is correlational, and like correlational solutions, there is nothing inherent in type II analysis to guarantee generality. Watson's reduction of thought to subvocalized speech is a form of type II analysis (Watson, 1924).

A type III analysis enumerates a set of components which are necessary, but not sufficient, for the presence of an element. If an element occurs, a specified set of components must occur, but the presence of components does not guarantee the presence of a higher order structure. As Polanyi (1966) has argued, a set of notes do not determine a musical composition, but are necessary for the construction of a composition. In this sense, a type III analysis may be identified with various biological and psychological theories of "emergence" which presume the occurrence of structures which are not strictly determined by their components (von Bertalanffy, 1968a, b; Waddington, 1970; Weiss, 1971). The term emergence in this context is

perhaps unfortunate since emergence implies an antecedent implicit structure which becomes manifest at a higher order. But in what sense is a composition contained in a keyboard?

A type IV analysis divides an element into a set of components which are necessary and sufficient for the presence of the element. A specified set of lower order components strictly determine a higher order structure. Since components strictly determine structure, no level-specific rules, principles, or properties are required to account for a structure as in a type III analysis. A single, interdependent set of assertions unambiguously determine composition and organization. A type IV analysis is amenable to strict formalization. As such, it is an ideal of scientific analysis. Formal necessity displaces empiricist and post hoc explanation.[1]

The four types of analysis applied to the decomposition of a stage represent a hierarchy of formal and scientific adequacy. A type I analysis provides no method to infer a lower order from a higher order or vice versa; a type II analysis proves an empirical approximation. A type III analysis proves a method to determine a lower order from a higher order, but not the obverse; a type IV analysis proves a complete solution. Type I and II analyses are relatively weak, inductive approaches and type III and IV analyses are relatively strong, deductive approaches to the problem of hierarchy, stage composition, and organization.

Types of transition. Like the principles, decision rules, or algorithms which determine decomposition, the principles, decision rules, or algorithms which determine succession may be classed as one of four types in terms of the restrictions placed upon the relation between successive elements and their components. The types of analyses which apply to decomposition and succession are homologous.

A type I analysis specifies an element and its components which are neither necessary nor sufficient for the occurrence of a successive element and its components. A type II analysis specifies an element and its components which are sufficient, but not necessary; a type III analysis, an element and its components which are necessary, but not sufficient; and a type IV analysis, an element and its components which are necessary and sufficient. Like the types of analysis which apply to decomposition, the types of analysis which apply to succession define a hierarchy of formal adequacy.

Infrastructural Attributes

The unitary sequences which may derive through the decomposition of elements are either simple or cumulative, conjunctive or disjunctive. These attributes describe a set of relationships between elements and implicitly a set of relationships between the components of elements. These relationships restrict the general organization and composition of elements.

The simple or cumulative, conjunctive or disjunctive characteristics or unitary sequences were defined in an earlier paper on general stage progressions (Van den Daele, 1969) and have been incorporated, in one form or another, in a variety of analyses on change and transition (Flavell, 1970, 1972; Looft, 1972; Reese, 1973; Riegel, 1969, 1972). In this section, the definitions of these attributes are recapitulated and elaborated to provide a framework for infrastructural analysis. The extended definitions complement earlier definitions, and with reference to the relation of elements add the notion of partial conjunction and disjunction.

Simple or cumulative. The elements of a simple sequence are exclusive (a V b). One and only one stage may characterize an individual at a time.

The elements of a cumulative sequence are nonexclusive (a ∧ b). More than one stage may characterize an individual at a time, but since ordered development implies change, at least some element of a cumulative sequence is discrete at successive intervals.

The components of discrete elements differ in constitution, organization, or both where constitution is the substance of components, and organization, the relation of components. Elements which are not discrete are common. The components of common elements are identical in constitution and organization.

Conjuctive or disjunctive. The relation between discrete elements is either partially or completely conjunctive or disjunctive. A conjunctive relation presumes the occurrence of some equivalencies or inclusions between discrete elements (a R b); and a disjunctive relation, their absence (a R' B).

If a component or set of components are common to a pair or more of discrete elements and no components are added or lost, the elements for which components are common are completely conjunctive.[2] If some components are common and some added or lost, the elements are partially conjunctive and partially disjunctive. If no components are common, the elements are completely disjunctive. The decomposition of elements into components obtains no common components other than the empty set.

Infrastructural Transition

Whether simple or cumulative, conjunctive or disjunctive, each sequence possesses a unique infrastructure. Infrastructure and transition are interdependent. Infrastructure is the static aspect of a developmental progression, and transition, its dynamic aspect. Infrastructure implies a

mode of transition, and transition, a type of infrastructure. The simple or cumulative, conjunctive or disjunctive properties of a progression do not occur in isolation, but in combination. The combination of these properties constrains the forms of admissible change from one element to another.

Simple or cumulative, conjunctive sequence. The elements of a conjunctive progression are transformed in a simple sequence, a, b, c... and correlatively transformed in a cumulative sequence, a, ab, abc.... A transformation or correlative transformation is a change in the organization or relation of components. When associated with a simple sequence, transformation is mutually exclusive; when associated with a cumulative sequence, it is nonexclusive, and hence correlative.

The common components of a conjunctive progression may be obtained at the same level of analysis, at successively lower levels, or successively higher levels. When common components occupy the same level, transformation or correlative transformation occurs with the parallelism of common components. When common components occupy successively lower or higher levels, transformation or correlative transformation occurs with the progressive subordination or emergence of common components.[3]

Within any level of analysis, a transformation or correlative transformation is a correspondence when components are in one-to-one relation; a differentiation, when components are in one-to-many relation; and an integration or concrescence when components are in many-to-one relation. By convention, a correspondence, differentiation, or integration describes the relations of components within the second level of analysis, although the form of intralevel transformation may be identified for any level if the level is specified.

The elements at the first level of analysis and parallel common components at any $1 + n$ level are always in correspondence. With these exceptions, any correspondence, differentiation or integration may occur with any parallelism, subordination, or emergence (Figure 3).

Insert Figure 3 about here

A conjunctive progression postulates common items which serve as "continuity components" in transformation. The occurrence of these components guarantees an intrinsic connection between successive stages while the relation of these components in subordinate or emergent progressions allows the partial ordering of stages in terms of intrinsic criteria.

The successive elements of a conjunctive progression, taken two at a time, are analogous to a set of balanced equations. For any transformation, there is an inverse, so given some element, its precursor or successor is obtainable by operation on its components, given some transformation rule.

Simple or cumulative, disjunctive progression. The elements of a disjunctive progression are substituted in a simple sequence, a, b, c... and added or deleted in a cumulative sequence, a, ab, abc.... A substitution, addition, or deletion is a change in the substance or constitution of components. A substitution is mutually exclusive, and an addition or deletion, nonexclusive. A substitution is an addition with a deletion.

Like the common components of a conjunctive progression, the discrete components of a disjunctive progression may occur at the same level of analysis, or at successively lower or higher levels. When components occupy the same level, substitution or addition occurs with the coplanarity of

discrete components, and when components occupy successively lower or higher levels, substitution or addition occurs with the progressive expansion or contraction of levels.

Within any level of analysis, a substitution or addition is coincident when components are in one-to-one relation; augmented, when components are in one-to-many relation, and reduced, when components are in many-to-one relation. Unless otherwise specified, a coincidence, augmentation, or reduction describes the relations of components within the second level of analysis.

The elements at the first level of analysis are always coincident. Otherwise, since no components are common and any element may contain any number of components and levels, any coincidence, augmentation, or reduction may occur with any coplanarity, expansion, or contraction (Figure 4).

Insert Figure 4 about here

A disjunctive progression postulates discrete components which derive through successive displacements. Since there are no common components, there is no transformation to obtain one element from another. A transformation implies an implicit identity.

In a disjunctive progression, elements are incommensurable. The items of analysis lack any substantive connection; hence, the assignment and order of elements obtain through extrinsic criteria. The relations between discrete components of disjunctive progressions mimic the relations between components of conjunctive progressions.

Hybrid progression. If in a progression some components are common and some are added or lost, the progression is a hybrid. A hybrid combines the

forms of infrastructure and transition which characterize conjunctive and disjunctive progressions. A hybrid is a partially conjunctive, partially disjunctive progression. In a hybrid, any transformation or correlative transformation may be qualified by any addition, deletion, or substitution of components within or between levels (Figure 5).

Insert Figure 5 about here

A hybrid progression combines common and discrete components and renders transformation and displacement incomplete. An incomplete transformation or its inverse yields more or less but not the same common components. A hybrid is problematic. Transformation is wedded to displacement, and substantive continuity to discontinuity. The successive elements of a hybrid progression taken two at a time yield a set of semibalanced equations.

The description and analysis of a hybrid requires the distinction of its conjunctive and disjunctive aspects by level of analysis. In this way, the substantively continuous and discontinuous may be placed into relation and the sources of transformation and displacement identified.

Decomposition and succession. A trivial decomposition may be associated with formal succession, or conversely, a formal decomposition with trivial succession. Nevertheless, certain forms of infrastructure and transition are so organized that what applies within elements applies between elements, and the rules of decomposition describe the rules of succession.

This relationship is summarized in the map theorem: If in some conjunctive progression, subordination is complete, that is, no subordinated components are added or lost, the subordinated components of the nth + 1 stage map the

components of the nth stage; and if emergence is complete, that is, no emergent components are added or lost, the emergent components of the nth stage map the components of the nth + 1 stage (Figure 6).

Insert Figure 6 about here

Formation and transformation are interdependent, and to identify one is to virtually identify the other. If subordination is complete, the proper object of study for the construction of developmental history is the terminal state and if emergence is complete, the initial state. The utility of either approach derives from the formal isomorphism of synchronic and diachronic organization.

Various theorems apply to type IV decomposition and type IV succession which do not apply to weaker forms of analysis. A set of unitary progressions obtained through a type IV decomposition with complete emergence cannot diverge from a common element or with complete subordination cannot converge. In either case, a common element maps identical components for all levels of analysis. Since the same rule of succession applies to all potential sequences, all potential sequences are isomorphic in composition from the point of divergence for an emergent progression, and to the point of convergence for a subordinate progression.

A conjunctive progression with complete parallelism of common components is not a type IV decomposition. When components are necessary and sufficient for the occurrence of a higher order element, identical components define identical elements. Hence the elements of a conjunctive progression with complete parallelism are identical, but a type IV progression with identical elements is a contradiction. At least some element must be discrete.

Although the majority of strong theorems apply to conjunctive sequences viewed from a formal perspective, at least some theorems apply to disjunctive sequences viewed from either formal or nonformal perspectives: If the elements of any progression are empty, the successive elements are added, deleted, or substituted. When elements are empty, the components of elements are not enumerable and there are no common components between elements.

The theorem of empty elements posits that certain progressions are inherently disjunctive and implies that demonstration of transformation requires decomposition. A large set of developmental sequences are empty in the sense described above, and, consequently evidence of order in development must largely derive through purely empirical observation. If a demonstration of transformation is desired, such elements must be redefined from another perspective or embedded in a larger context of analysis.

Infrastructural Illustrations

A developmental analysis of infrastructure and transition poses the question: What constitutes the changes in organization that characterize some progression? The question presumes the investigator apply a consistent perspective within given boundary conditions to some domain of interest. Unfortunately, too often stage theorists appear to have cultivated the habit of mixing levels of abstraction with pieces of content and bits of structure. Oftentimes the result possesses little structural consistency.

A kindred difficulty derives in developmental analyses concerned with surface structure. As argued earlier, evidence of transformation requires decomposition. Without some differentiation of surface structure, development takes on the appearance of a kind of quantum hopscotch.

Relatively few developmental theories are satisfactory theories of transformation. With the exception of explicitly cognitive and linguistic analyses of development (Brown, 1972; Pascual-Leone, 1966; Witz, 1971), the preponderance of current developmental analyses stress the discrete aspects of change with little concern for the identification of common components or their reorganization. Consequently, unambiguous representations of infrastructure and transition are confined to that relatively small set of theories which render explicit the constitution and organization of successive stages.

Conjuctive Progressions

A simple or cumulative conjunctive progression obtains through a transformation or correlative transformation with intralevel correspondence, integration, or differentiation, and interlevel parallelism, subordination or emergence. Within this group, one example is cognitive and one is linguistic. Both examples possess formal algorithms for the decomposition of wholes into parts and specify the common components in transformation.

Simple sequence. The Piagetian stages of cognitive development represent a type IV decomposition, type IV succession, complete subordinate progression with successive integration, then differentiation. The components of stage a, sensory-motor schema, are coordinated to yield the components of stage b, a set of preoperational actions and intuitions, which concresce to produce the components of stage c, a set of concrete operational groupings. Finally, concrete operational groupings, through logical multiplication, generate the components of stage d, the set of formal operational binary propositions (Figure 7).[4]

Insert Figure 7 about here

In a Piagetian sequence each successive stage reorganizes through equilibration of the components of precursor stages. Later stages incorporate earlier stages, "the inferior becomes part of the superior [Inhelder, 1956, p. 85]." The subordinated components of the nth + 1 stage map the components of the nth stage. Piaget's developmental theory is a synchronic theory turned on its side.

Cumulative sequence. McNeill's (1966) developmental analysis of the pivot class corresponds to a type IV decomposition, type IV succession, emergent progression with differentiation. The element a, at time 1, is the pivot class and includes an implicit hierarchy of class differentiation obtained through a purely formal analysis of potential tree structure. The element b, at time 2, is composed of the grammatical classes of articles and demonstratives; and the element c, at time 2, is the pivot class obtained through class subtraction of the components of b from a. The element b, at time 3, is identi to b at time 2; that is, the child's competence to pose and unpose articles and demonstratives is retained. The element d, at time 3, includes the grammatical classes of adjectives and possessives while the element e, at time 3, is the new pivot class derived through class subtraction of the components of d from c (Figure 8).

Insert Figure 8 about here

In a McNeill sequence, the decomposition of elements of an earlier stage yields the components of later stages. Implicit grammatical classes

become explicit. The emergent components of the nth stage map the components of the nth + 1 stages. In McNeill's words, "There is no evidence of independent discovery of the adult grammatical classes; they are merely removed from the pivot class like a banana peel [1966, p. 28]." Like Piaget's theory, McNeill's sequence is a synchronic theory turned on its side, but in another direction.

Disjunctive Progressions

A simple or cumulative disjunctive progression obtains through substitution, addition, or deletion of discrete elements with intralevel coincidence, augmentation, or reduction, and interlevel coplanarity, expansion, or contraction. Within this group, one example is derived from a learning perspective and another, from a maturational perspective. The selection of examples may appear paradoxical insofar as the learning and maturational perspectives stress discrete bases of behavior change. Yet the perspectives, as epitomized in classical learning theories and psychometric evaluations of social or mental age, share an astructural bias. In the examples cited, the emphasis is upon the discreteness of successive behaviors with little or no emphasis upon their intrinsic relation.

Simple sequence. A plausible Bandura and Walters (1963) model of behavior development corresponds to a type II decomposition, type I succession, expansive progression with augmentation. While the mode of decomposition is empirical, the relation between successive behaviors is essentially trivial, that is, a precursor behavior is neither necessary nor sufficient for a successor behavior. While Bandura and Walters provide no specific behavioral stages for infrastructural analysis (which would be inconsistent with a theory of trivial

succession), differential exposure to models with more varied behavioral repertoires linked to successive periods of socialization yield changes in the complexity of modeled behavior (Figure 9).

Insert Figure 9 about here

In classical and social learning analyses of behavior development, behavior change is presumed to obtain through largely external influences such as the pattern of reinforcement or juxtaposition of stimuli (Gewirtz, 1969). An earlier response is extinguished (deleted) and a later response established (added). Since the relation between successive behaviors is neither necessary nor sufficient, invariance of behavior development is denied, ignored, or imputed to maturation. Sequence, from a learning perspective, is specific to learning history or experimental manipulation.

Cumulative sequence. Frankenburg and Dodds' (1967) Denver Developmental Screening Test corresponds to a type III decomposition, type II succession, expansive progression with augmentation. The mode of decomposition is type III since certain minimum characteristics of a response are necessary to qualify it at a given level of maturity. The mode of succession is type II since some set of precursor responses (a minimum performance at an earlier normative level) suggest or imply, but do not determine or necessitate, some set of successor responses (a minimum performance at a later normative level). The Denver test qualifies as cumulative in two senses: First, the various categories of behavior, gross motor, fine motor, linguistic, and personal-social, are not exclusive, and second, within any category, earlier and later behaviors may coexist. If in the Denver test each successor item is,

in some sense, more complex and hierarchized than its predecessor, transition implicates expansion with augmentation (Figure 10).

Insert Figure 10 about here

The Denver test in particular, and normative examinations in general, stress the discreteness of age-dependent responses. Discreteness augments discriminability, and discriminability augments reliability. Discreteness is a matter of psychometric expediency.

Hybrid Progressions

A hybrid progression obtains through a transformation or correlative transformation with some substitution, addition, or deletion. Within this group, one example describes personality development and one describes emotional development. The respective theories presume an earlier stage conditions or affects a later stage, although a later stage is not a simple outgrowth of an earlier stage, but requires, in some sense, an incorporation of new information or modes of organization.

Simple sequence. Erikson's (1963) analysis of ego development as a set of choice points associated with a set of personal-cultural dilemmas represents a type II decomposition, type III succession hybrid progression with an unspecified infrastructural form. The diffuseness of definition which characterizes Erikson's stages renders a large set of characteristics sufficient, but not necessary, to place a subject more or less at some stage. Nevertheless, Erikson maintains that some solution to each stage is necessarily antecedent to a subsequent stage. Whatever antecedent solution is obtained affects, in some way, the form of a subsequent solution. Earlier choices are amalgamated or coordinated

to later choices. "Basic mistrust" once obtained modifies later "autonomy," "initiative," etc.

The intuitive form of Erikson's stages renders a rigorous formal analysis difficult. Similar analytic problems are associated with kindred clinical progressions. From a formal perspective, a major advance in such intuitive theories would derive through the identification of common and discrete components at successive stages wi h some representation of their mutual relation.

Cumulative sequence. Bridges' classical description of emotional development may be characterized as a type II decomposition, type III succession, cumulative hybrid progression with a partially specified infrastructure (Bridges, 1931, 1932). In Bridges' scheme, a given emotional response possesses alternative age-specific characteristics while latter patterns of response derive, in part, from earlier patterns. The response components of Bridges' "original emotion" generalized excitement, coordinate with new response components, associated with maturation and learning, to yield the later emotions, distress and delight and their derivatives (1932, p. 325). Later acquisitions do not displace earlier acquisitions, but augment emotional expression.

Bridges' scheme of emotional development is an essentially descriptive statement of behavior patterns associated with types of affect at successive ages. The postulated relation of emotions is suggested by selected similarities with little or no formal analysis or representation of the coordination of earlier to latter affective expression. This nonformal perspective renders ambiguous derivation of certain classes of early expression. In what sense,

for example, does quiescent smiling as well as fearful protest arise from undifferentiated excitement (1932, p. 334)? As with Erikson's ego theory, an identification of the common and discrete components of development might reduce ambiguity and enhance utility.

Infrastructural Evaluation

The description of a developmental sequence implies identification of a domain, adoption of a perspective, definition of boundary conditions and adoption of a mode of decomposition and succession. Taken together, these attributes of analysis constrain the forms of infrastructure and transition admissible in a sequence. A change in any one attribute redefines the analytic task. The attributes of analysis are interdependent.

The interdependence of attributes and their logical-hypothetical foundation implies the relativism of developmental analysis. Change is as change is defined. As with a meatgrinder, what is placed at one end emerges at the other. Nevertheless, as with meat and meatgrinders, some representations of infrastructure and transition are better than others.

As argued in earlier sections, a developmental analysis derived from a common perspective applied to all elements of a progression is better than one derived from discrete perspectives, and a type of decomposition and succession which requires a relation of necessity and sufficiency between items of analysis is better than alternative types of decomposition and succession.

Given a consistent perspective and comparable type of decomposition and succession, a conjunctive progression is preferable to a disjunctive one. The elements of a conjunctive progression are unequivocally members of a

common set, but the elements of disjunctive progression are incommensurable. As with the null hypothesis, a disjunctive progression is not conclusive since from some alternative perspective, the discrete may be construed as common.

Yet displacement and substitution, the discontinuous in development, must be admitted as a potential class of change. Human growth and development, unlike the growth and development of cell cultures in an ideal medium, may not proceed in a state-determined way, but hybridize the predisposed with the serendipitous. An open system admits perturbations which may modify or direct the course of change. Development may not yet be reduced to a special case of a Laplacian universe.

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Footnotes

[1]In a type IV analysis, if components within and between any nth and nth + 1 level are discrete, that is, no two are identical, and if there are n components at the nth level, then at the nth + 1 level, there is a minimum number of subcomponents, m :

$$m = \frac{n!}{[(n-2)!2!]}$$

providing no component is empty, no subcomponent is repeated, and the order of combination of subcomponents is irrelevant in the formation of higher order components. If the order of combination is relevant and the same restrictions apply, there is a minimum number of subcomponents, m'

$$m' = \frac{n!}{(n-2)!} \quad .$$

Similar formulae may be derived for conditions which allow repetition of subcomponents in the composition of components, or adjustments may be made for situations where no component is composed of less than three subcomponents, and so on. The point is that type IV analyses allow the strict determination of relationships between levels of analysis given certain presuppositions dependent upon perspective.

[2]When discrete elements obtain the same components through iteration, the level at which the common components are obtained is the base level of the element. Although such components may be further decomposed, if the decomposition is consistently applied, the enumeration of subcomponents of common components is redundant.

[3]When common components occur at mixed levels, transformation occurs with partial parallelism, partial subordination, or partial emergence. To simplify nomenclature, terms may be hyphenated and the qualifier "partial" deleted. Hence, common components at mixed levels occur in a parallel-subordinate, parallel-emergent, subordinate-emergent or parallel-subordinate-emergent relation.

[4]An alternative interpretation of formal operations emphasizes the INRC group properties to which the 16 binary propositions are subordinated. If this interpretation is adopted, the transition from concrete to formal operations is an integration, not a differentiation.

Figure Captions

Fig. 1. Partition of a multiple sequence. The number of unitary sequences contained within a multiple progression is equivalent to the maximum number of pathways between successive elements for that progression.

Fig. 2. Transposition of a decomposition. In the example, an element of some progression, represented by the "a" at the first level of analysis, is decomposed into two components, the "bs" at the second level of analysis, which, in turn, are decomposed into four components, the "cs" at the third level of analysis. Components at successive levels are obtained through application of some algorithm of decomposition. The transposition from "branch" form to a "linear hierarchical" form is a notational convenience.

Fig. 3. Infrastructure and transition in representative conjunctive sequences. The decomposition of elements allows the identification of "continuity components" in transformation. A transformation implies a common substrate.

Fig. 4. Infrastructure and transition in representative disjunctive sequences. The decomposition of an element reveals no common components at any level of analysis. A substitution implies a discrete substrate.

Fig. 5. Infrastructure and transition in representative hybrids. The decomposition of elements reveals a partial identity of components.

Fig. 6. Map theorem. In complete emergent or subordinate progressions, synchronic and diachronic organization are isomorphic.

Fig. 7. Piaget's cognitive stages. Only the general form of cognitive integration, then differentiation is represented. A complete representation requires the enumeration of a larger subset of components.

Fig. 8. McNeill's developmental analysis of the pivot class. Earlier competence to pose and unpose grammatical classes coexists with later competence.

Fig. 9. An imitative sequence. The representation presumes increased complexity with the extinction or displacement of earlier modes of behavior.

Fig. 10. Normative sequence. A large set of items on normative inventories are selected to enhance discriminability, hence discreteness.

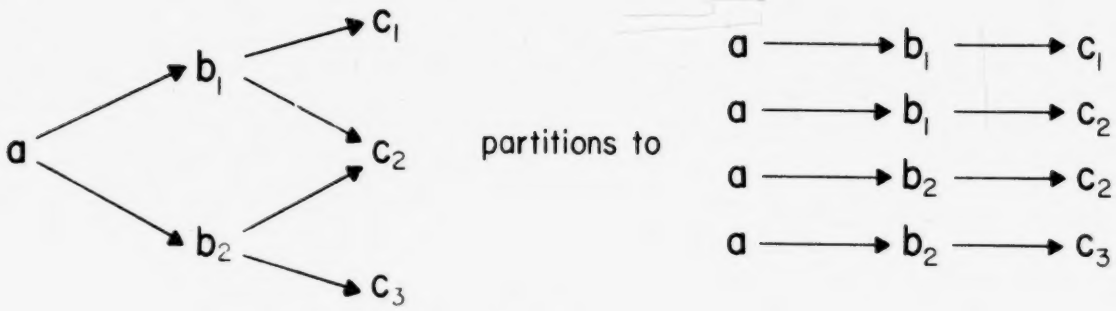

Fig. 1

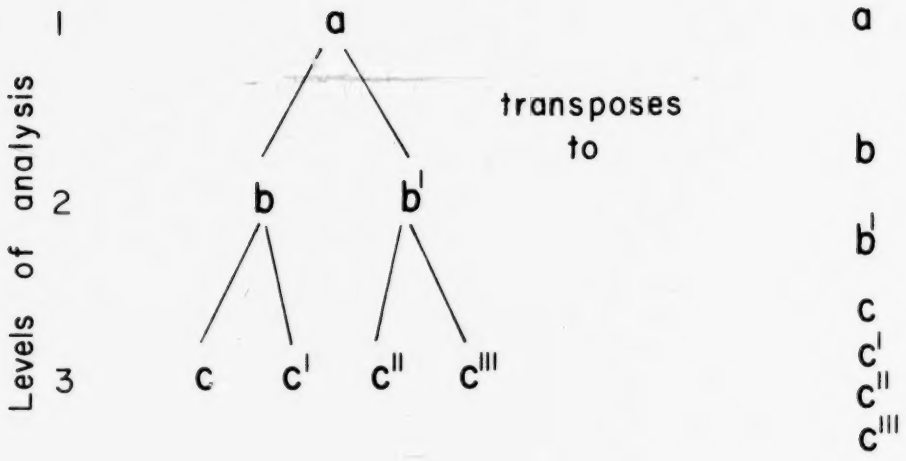

Fig. 2

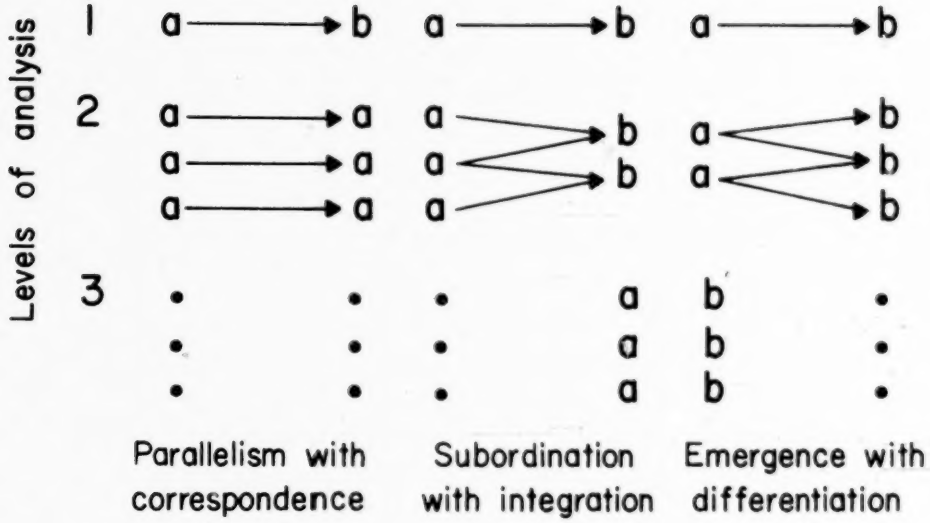
Levels of analysis
1
2
3
a
b
Parallelism with correspondence
Subordination with integration
Emergence with differentiation

Fig. 3

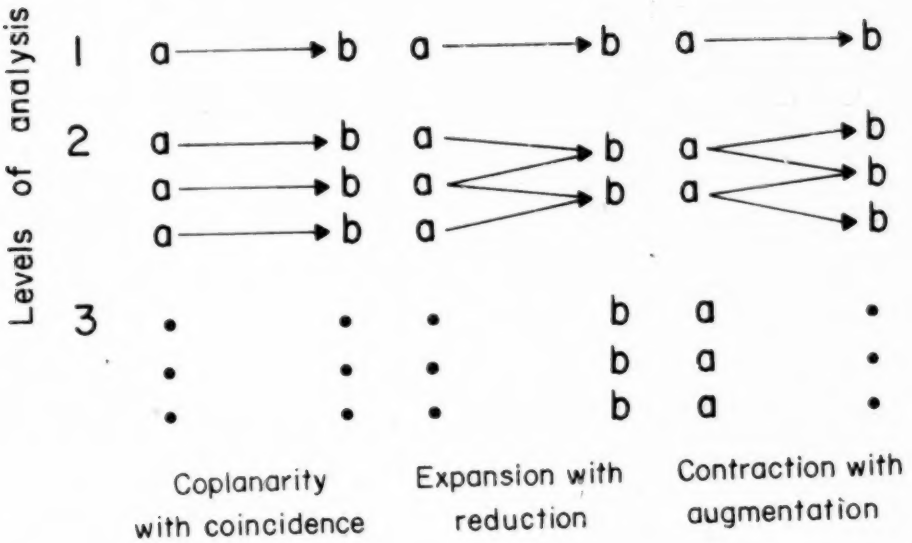
Levels of analysis
1
2
3
a
b
Coplanarity with coincidence
Expansion with reduction
Contraction with augmentation

Fig. 4

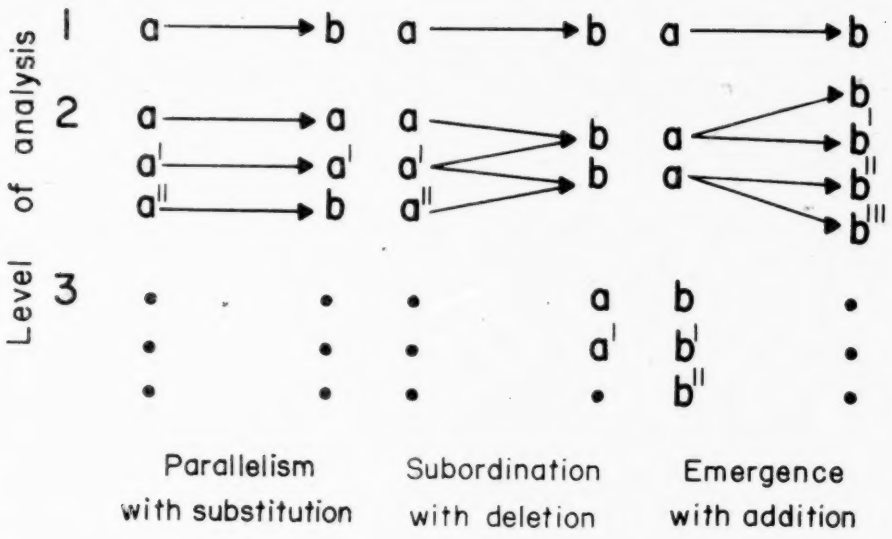
Level of analysis
1
2
3
a → b
a → a
a' → a'
a'' → b
a → b
a' → b
a'' → b
a
a'
b
b'
b''
a → b
a → b'
a → b''
a → b'''
Parallelism with substitution
Subordination with deletion
Emergence with addition

Fig. 5

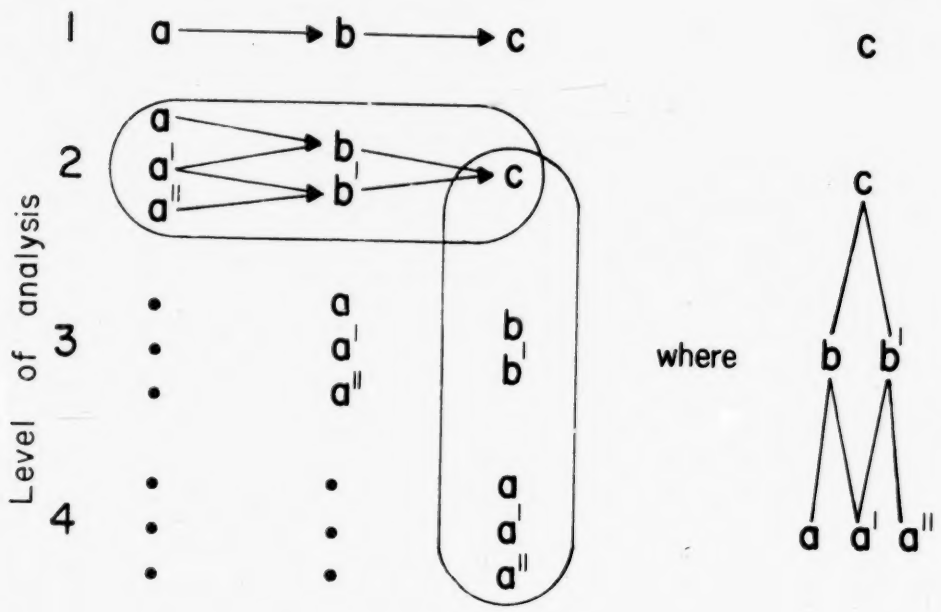

Fig. 6

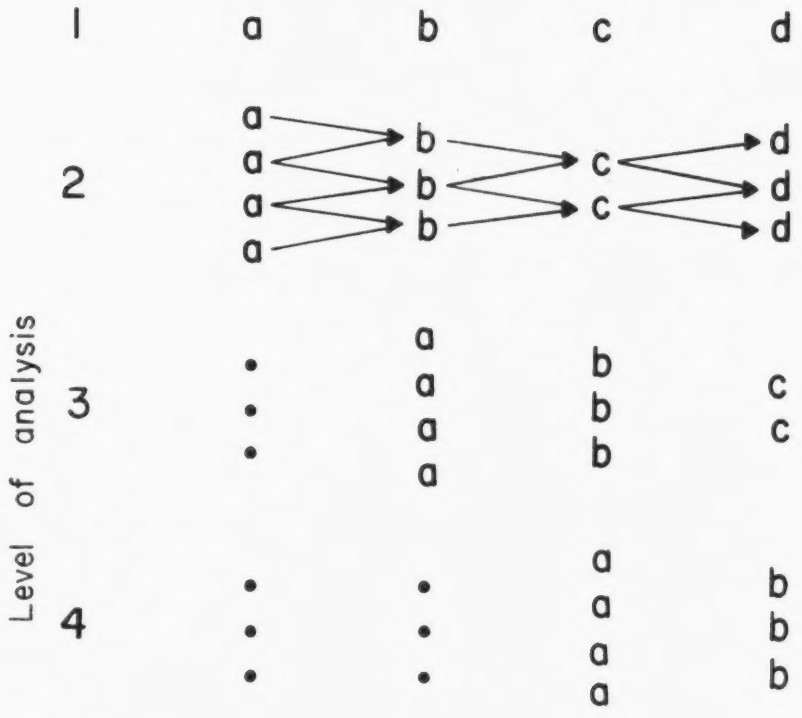

Fig. 7

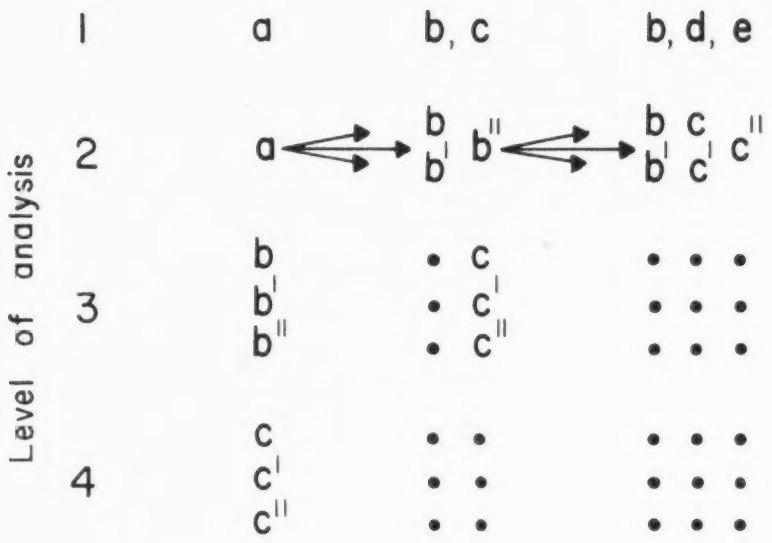

Fig. 8

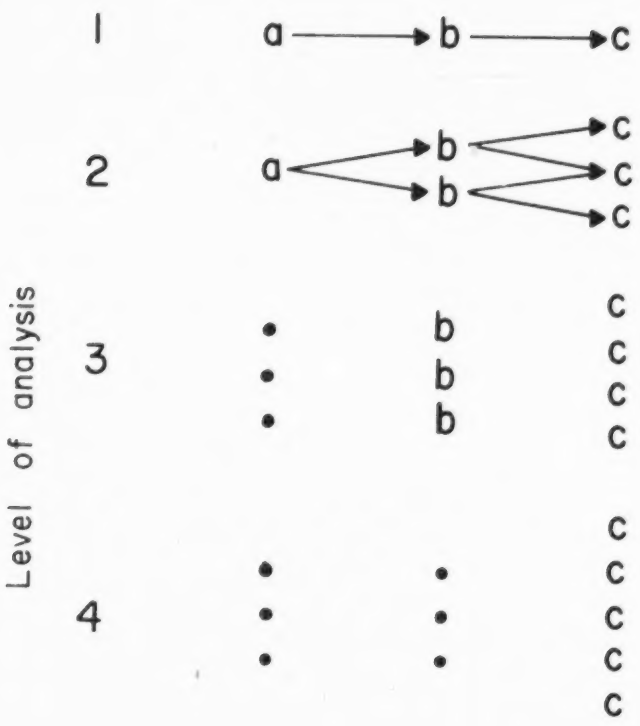

Fig. 9

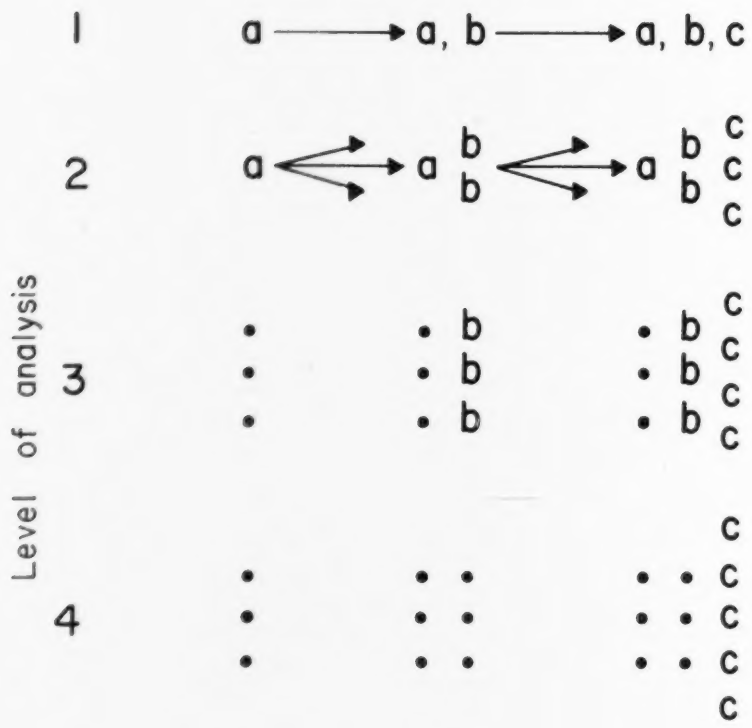

Fig. 10